UNDERSTANDING LANGUAGE/
STANFORD CENTER FOR ASSESSMENT, LEARNING, AND EQUITY

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Principles for the Design of Mathematics Curricula: Promoting Language and Content Development

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February 28, 2017
Version 2.0

Acknowledgements

The authors wish to express their gratitude to several colleagues who contributed greatly to this project. First and foremost, we would like to thank Kenji Hakuta for his dedication and guidance. Without his leadership and commitment this project would not have been possible. The project also benefitted from the discussions, iterative feedback cycles, and general support provided by Maura Dudley, Rebecca Bergey, and Magda Chia.

Resources produced by UL/SCALE at Stanford University are available electronically at http://ell.stanford.edu.

Recommended citation:

Zwiers, J., Dieckmann, J., Rutherford-Quach, S., Daro, V., Skarin, R., Weiss, S., & Malamut, J. (2017). *Principles for the Design of Mathematics Curricula: Promoting Language and Content Development*. Retrieved from Stanford University, UL/SCALE website: http://ell.stanford.edu/content/mathematics-resources-additional-resources

OVERVIEW

The purpose of this document is to nudge the field forward by offering support to the next generation of mathematics learners and by challenging persistent assumptions about how to support and develop students' disciplinary language. Our goal is to provide guidance to mathematics teachers for recognizing and supporting students' language development processes in the context of mathematical sense making. We provide a framework for organizing strategies and special considerations to support students in learning mathematics practices, content, and language. The framework is intended to help teachers address the specialized academic language demands in math when planning and delivering lessons, including the demands of reading, writing, speaking, listening, conversing, and representing in math (Aguirre & Bunch, 2012). Therefore, while the framework can and should be used to support *all students* learning mathematics, it is particularly well-suited to meet the needs of linguistically and culturally diverse students who are simultaneously learning mathematics while acquiring English.

OUR THEORY OF ACTION

Systemic barriers for language learners persist not only in tasks and materials, but in educators' presentational language, expectations for peer interactions, and assessment practices. Only through intentional design of materials, teacher commitments, administrative support, and professional development can language development be built into teachers' instructional practice and students' classroom experience. Our theory of action for this work is grounded in the **interdependency** of language learning and disciplinary learning, the central role of **student agency** in the learning process, the importance of **scaffolding routines** that foster students' independent participation, and the value of **instructional responsiveness** in the teaching process.

Mathematical understandings and language competence develop interdependently. Deep disciplinary learning is gained through language, as it is the primary medium of school instruction (Halliday, 1993). Ideas take shape multi-modally, through words, texts, illustrations, conversations, debates, examples, etc. Teachers, peers, and texts serve as language resources for learning (Vygotsky, 1978). Content teachers (implicitly/explicitly) teach the language of their discipline. Instructional attention to this language development, historically limited to vocabulary instruction, has now shifted to also include instruction around the demands of argumentation, explanation, analyzing purpose and structure of text, and other disciplinary discourse.

Students are agents in their own mathematical and linguistic sense-making. One prevailing assumption is that mathematical language proficiency means using only formal definitions and vocabulary. Although that is how math is often more formally presented in textbooks, this type of language does not reflect the process of exploring and learning mathematics. Another common assumption is that developing the language of the discipline requires continuous “time-outs” from the content, and multiple detours into “math language” mini-lessons. However, through successive and supportive experiences with math ideas, learners make sense of math with their **existing** language toolkit (Moschkovich, 2012), while also expanding their language repertoire with tools and mathematics conventions as they come to see these tools (e.g., definitions, properties, procedures) as useful in accomplishing a meaningful goal.[1]

We challenge both of these assumptions because we see “language as action” (van Lier & Walqui, 2012): in the very doing of math, students have naturally occurring opportunities to learn and notice mathematical ways of making sense and talking about ideas and the world. It is our responsibility as educators to structure, highlight, and bolster these opportunities, making explicit the many different ways that mathematical ideas are communicated, rather than acting as “the keepers’” or “the givers” of language. A commitment to help students develop their own command of the “mathematical register” is therefore not an additional burden on teachers, but already embedded in a commitment to supporting students to become powerful mathematical thinkers and ‘do-ers’ (Lee, Quinn & Valdés, 2013).

Scaffolding provides temporary supports that foster student autonomy. Some educators hold a more traditional assumption that students will learn the English language **and** disciplinary language by merely being immersed in them over time, with little additional support. This presents serious equity and access issues that cannot go unchallenged. Disciplinary language development occurs when students use their developing language to make meaning and engage with challenging disciplinary content understandings that are beyond students’ mathematical ability to solve independently. However, these tasks should include temporary supports that students can use to make sense of what is being asked of them and to organize their thinking. Learners with emerging language – at any level – can engage deeply with central disciplinary ideas under specific instructional conditions (Walqui & van Lier, 2010). Temporary supports, or scaffolds, can include teacher modeling, supporting students in making charts with mathematical information from a word problem, providing manipulatives or graphic organizers to support sense-making, identifying and drawing upon students’ inner resources, and structured peer interactions. Immediate feedback from intentionally-designed peer interaction helps students revise and refine not only the way they organize and communicate their own ideas, but also the way they ask questions to clarify their understandings of others’ ideas.

[1] A meaningful goal might be explaining a problem solving technique, modeling a solution, or justifying an argument.

Instruction supports learning when teachers respond to students' verbal and written work. The mathematics language routines described later in this document help both teachers and students keep one eye and ear on language as much as possible, focusing attention to student language that support in-the-moment teacher, peer, and self-assessment (Cazden, 2001). Teachers can adapt and respond more effectively to what students are saying and doing as they develop disciplinary language and content understanding concurrently.

Based on their observations of student language, teachers can make adjustments to their teaching and provide additional language scaffolding where necessary. Teachers can select from the "heavier" or "lighter" supports provided in the curriculum as appropriate. When selecting from these supports, teachers should take into account the language demands of the task in relation to their students' English language proficiency.

OUR FRAMEWORK

This framework includes **four design principles** for promoting mathematical language use and development in curriculum and instruction. The design principles and related routines work to make language development an integral part of planning and delivering instruction while guiding teachers to amplify the most important language that students are expected to bring to bear on the central mathematical ideas of each unit. The design principles, elaborated below, are:

Design Principle 1: Support sense-making
Design Principle 2: Optimize output
Design Principle 3: Cultivate conversation
Design Principle 4: Maximize linguistic and cognitive meta-awareness

These four principles are intended as guides for curriculum development and planning and execution of instruction, including the structure and organization of interactive opportunities for students, and the observation, analysis, and reflection on student language and learning. The design principles motivate the use of **mathematical language routines,** described in detail below, with examples. The eight routines included in this document are:

MLR1: Stronger and Clearer Each Time
MLR2: Collect and Display
MLR3: Critique, Correct, and Clarify
MLR4: Information Gap
MLR5: Co-Craft Questions and Problems
MLR6: Three Reads
MLR7: Compare and Connect

MLR8: Discussion Supports

DESIGN PRINCIPLES FOR PROMOTING MATHEMATICAL LANGUAGE USE AND DEVELOPMENT IN CLASSROOMS

Principle 1

SUPPORT SENSE-MAKING: *Scaffold tasks and amplify language so students can make their own meaning.*

Students do not need to understand a language completely before they can start making sense of academic content and negotiate meaning in that language. Language learners of all levels can and should engage with grade-level content that is appropriately scaffolded. Students need multiple opportunities to talk about their mathematical thinking, negotiate meaning with others, and collaboratively solve problems with targeted guidance from the teacher (Cazden, 2001; Moschkovich, 2013). In addition, teachers can foster students' sense-making by amplifying rather than simplifying, or watering down, their own use of disciplinary language.

Teachers should make language more "considerate" to students by amplifying (Walqui & van Lier, 2010) rather than simplifying speech or text. Simplifying includes avoiding the use of challenging texts or speech. Amplifying means anticipating where students might need support in understanding concepts or mathematical terms, and providing multiple ways to access those concepts and terms. For example, organizing information in a clear and coherent way, providing visuals or manipulatives, modeling problem-solving, engaging in think-alouds, creating analogies or context, and layering meaning are all ways to amplify teacher language so that students are supported in taking an active role in their own sense-making of mathematical relationships, processes, concepts and terms.

Several routines can be used to Support Sense-Making. In particular, MLR2 - Collect and Display, MLR6 – Three Reads, and MLR8 – Discussion Supports.

Principle 2

OPTIMIZE OUTPUT: *Strengthen the opportunities and supports for helping students to describe clearly their mathematical thinking to others, orally, visually, and in writing.*

Linguistic **output** is the language that students use to communicate their ideas to others. Output can come in various forms, such as oral, written, visual, etc. and refers to all forms of student linguistic expressions except those that include significant back-and-forth negotiation of ideas. (That type of *conversational* language is addressed in the third principle.)

Students need repeated, strategic, iterative and supported opportunities to articulate complex mathematical ideas into words, sentences, and paragraphs (Mondada, 2004). They need spiraled practice in (a) making their ideas stronger with more robust reasoning and examples, and (b) making their ideas clearer with more precise language and visuals. They need to make claims, justify claims with evidence, make conjectures, communicate their reasoning, critique the reasoning of others, and engage in other mathematical practices. Increasing the quality and quantity of opportunities to describe mathematical reasoning also will allow teachers to frequently formatively assess students' content learning and language use so that teachers can provide feedback and differentiate instruction more effectively.

Several routines can be used to Optimize Output. In particular, MLR1 – Stronger and Clearer, MLR3 – Critique, Correct, and Clarify, MLR4 – Info Gap, MLR5 – Co-craft Questions and Problems, and MLR7 – Compare and Connect.

Principle 3

CULTIVATE CONVERSATION: *Strengthen the opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class).*

Conversations are back-and-forth interactions with multiple turns that build up ideas about math. Conversations act as scaffolds for students developing mathematical language because they provide opportunities to simultaneously make meaning and communicate that meaning (Mercer & Howe, 2012; Zwiers, 2011). They also allow students to hear how other students express their understandings. When students have a reason or purpose to talk and listen to each other, interactive communication is more authentic. For example, when there is an "information gap," in which students need or want to share their thoughts (which are not the same), students have a reason or purpose in talking and listening to each other.

During effective discussions, students pose and answer questions, clarify what is being asked and what is happening in a problem, build common understandings, and share experiences relevant to the topic. As mentioned in Principle 2, learners must be supported in their use of language, including within conversations, to make claims, justify claims with evidence, make conjectures, communicate reasoning, critique the reasoning of others, and engage in other mathematical practices – and above all, to make mistakes. Meaningful conversations depend on the teacher using lessons and activities as opportunities to build a classroom culture that motivates and values efforts to communicate.

Many routines can be used to Cultivate Conversation. In particular, MLR1 – Stronger and Clearer, MLR3 – Critique, Correct, and Clarify, MLR4 – Info Gap, MLR5 – Co-craft Questions and Problems, MLR7 – Compare and Connect, and MLR8 – Discussion Supports.

Principle 4

MAXIMIZE META-AWARENESS: *Strengthen the "meta-" connections and distinctions between mathematical ideas, reasoning, and language.*

Language is a tool that not only allows students to communicate their math understanding to others, but also to *organize* their own experience, ideas, and learning for themselves. **Meta-awareness** is consciously thinking about one's own thought processes or language use. Meta-awareness develops when students and teachers engage in classroom activities or discussions that bring explicit attention to what students need to do to improve communication and/or reasoning about mathematical concepts. When students are using language in ways that are purposeful and meaningful for themselves, in their efforts to understand – and be understood by – each other, they are motivated to attend to ways in which language can be both clarified and clarifying (Mondada, 2004).

Meta-awareness in students is strengthened when, for example, teachers ask students to explain to each other the strategies they brought to bear to solve a challenging multi-step problem. They might be asked, "How does yesterday's method connect with the method we are learning today?," or, "What ideas are still confusing to you?" These questions are **metacognitive** because they help students to reflect on their own and others' learning of the content. Students can also reflect on their expanding use of language; for example, by comparing the differences between how an idea is expressed in their native language and in English. Or by comparing the language they used to clarify a particularly challenging mathematics concept with the language used by their peers in a similar situation. This is called **metalinguistic** because students reflect on English as a language, their own growing use of that language, as well as on particular ways ideas are communicated in mathematics. Students learning English benefit from being aware of how language choices are related to the purpose of the task and the intended audience, especially if an oral or written report is required. Both the metacognitive and the metalinguistic are powerful tools to help students self-regulate their academic learning and language acquisition.

Many routines can be used to develop and formatively assess students' meta-cognitive and meta-linguistic awareness. In particular, MLR2 - Collect and Display, MLR3 – Critique, Correct, and Clarify, MLR5 – Co-craft Questions and Problems, MLR6 – Three Reads, MLR7 – Compare and Connect, and MLR8 – Discussion Supports, lend themselves well to having students place extra attention on the language used to engage in mathematical communication and reasoning.

MATHEMATICAL LANGUAGE ROUTINES

The following mathematical language development routines were selected because they are the most effective and practical for simultaneously learning mathematical practices, content, and language. These routines also can be used in most lessons and across grade levels. **A 'math language routine' refers to a structured but adaptable format for amplifying, assessing, and developing students' language**. The routines emphasize the use of language that is meaningful and purposeful, not inauthentic or simply answer-based. These routines can be adapted and incorporated across lessons in each unit to fit the mathematical work wherever there are productive opportunities to support students in using and improving their English and disciplinary language.

These routines facilitate attention to student language in ways that support in-the-moment teacher-, peer-, and self- assessment. The feedback enabled by these routines will help students revise and refine not only the way they organize and communicate their own ideas, but also ask questions to clarify their understandings of others' ideas.

Mathematical Language Routine 1: Stronger and Clearer Each Time

Purpose: To provide a structured and interactive opportunity for students to revise and refine both their ideas and their verbal and written output (Zwiers, 2014). This routine provides a purpose for student conversation as well as fortifies output. The main idea is to have students think or write individually about a response, use a structured pairing strategy to have multiple opportunities to refine and clarify the response through conversation, and then finally revise their original written response. Throughout this process, students should be pressed for details, and encouraged to press each other for details. Subsequent drafts should show evidence of incorporating or addressing new ideas or language. They should also show evidence of refinement in precision, communication, expression, examples, and/or reasoning about mathematical concepts.

Example 1 – Successive Pair Shares

1. PRE-WRITE: Have students, individually, look at a problem and write down

their idea/reasoning for solving the problem a certain way, or any thoughts or questions about it, in complete sentences if possible. This is the pre-write sample; there will be a post-write to see if the sharing with others makes a difference. [Optional scaffold: Provide part of an initial draft for students to begin with that contains the language needed for an important idea.]

2. THINK TIME: Then give a minute for students to think about what they will say to the first partner to explain what they are doing, or did, to solve it. (They can't look at what they wrote while talking).
3. STRUCTURED PAIRING: Use a successive pairing structure. (For example: Have students get into groups of 6 or 8, with inner circles of 3 or 4 facing outer circles of 3 or 4). Remind students that oral clarity and explaining reasoning are important. Even if they have the right answer or they both agree, the goal is either (1) to be able to clearly explain it to others as a mathematician would or (2) for the other person to truly understand the speaker's ideas. Goal (1) is appropriate when students are further along in the development of a concept; goal (2) is appropriate closer to when students are first introduced to a concept.
4. IN PAIRS: When one partner is listening, he or she can ask clarifying questions, especially related to justifying (Why did you do that?). The other person then also shares and the listener also asks clarifying questions to draw more language and ideas out of quiet partners, if needed.
5. SWITCH: Partners switch one, two, or three more times, strengthening and clarifying their idea each time they talk to a new partner. Optionally, turns can emphasize strength (focus on math concepts and skills) or clarity (how to describe the math to others). Scaffolds can be removed with each successive pairing to build student independence.
6. POST-WRITE: Have students return to seats and write down their final explanations, in sentences (they can use drawings, too, explained by sentences). Turn in.

Example 2 – Convince Yourself, a Friend, a Skeptic

Students create three iterations of a mathematical argument or justification for three different audiences.

1. For the first draft, students explain or justify their argument in whatever way initially makes sense to them.
2. In the second draft, students are encouraged to explain WHAT they know and HOW they know it is true. Their explanations should include words, pictures, and numbers. They trade their written arguments with a peer who acts as a "friend" giving feedback on these components (WHAT and HOW).
3. In the third draft, students are encouraged to explain WHY what they know is

true by supporting their claims with evidence. Their explanations should include words, pictures, numbers, and examples. They should include examples that look like they might not be true but actually are. They should anticipate and address counter-arguments. They trade their written arguments with a peer who acts as a "skeptic" giving feedback on these components (WHY, examples, counter-arguments).

Mathematical Language Routine 2: Collect and Display

Purpose: To capture students' oral words and phrases into a stable, collective reference. The intent of this routine is to stabilize the fleeting language that students use in order for their own output to be used as a reference in developing their mathematical language. The teacher listens for, and scribes, the language students use during partner, small group, or whole class discussions using written words, diagrams and pictures. This collected output can be organized, revoiced, or explicitly connected to other language in a display that all students can refer to, build on, or make connections with during future discussion or writing. Throughout the course of a unit, teachers can reference the displayed language as a model, update and revise the display as student language changes, and make bridges between student language and new disciplinary language. This routine provides feedback for students in a way that increases sense-making while simultaneously supporting meta-awareness of language.

Example 1 – Gather and Show Student Discourse (Dieckmann, 2017)

During pair/group work, circulate and listen to student talk during pair work or group work, and jot notes about common or important words and phrases, together with helpful sketches or diagrams. Scribe students' words and sketches on visual display to refer back to during whole class discussions throughout the unit. Refer back to these words, phrases, and diagrams by asking students to explain how they are useful, asking students to clarify their meaning, and asking students to reflect on which words and visuals help to communicate ideas more precisely.

Example 2 – Number Talks (Humphreys & Parker, 2015)

1. INDEPENDENT THINK: Present students with a numeracy problem to be solved without paper for 1-2 minutes
2. WHOLE CLASS SHARE-OUT: Have students share the method or strategy they used to arrive at an answer
3. DISPLAY STUDENT IDEAS: As students share their strategies, create a visual display for each of their methods or have students create their own

visual displays

4. ASK PROBING QUESTIONS: Ask students to compare and contrast the displayed methods (See MLR7), the benefits and drawbacks of displayed methods in different contexts, and/or to apply a certain student's method to a new problem.

Mathematical Language Routine 3: Critique, Correct, and Clarify

Purpose: To give students a piece of mathematical writing that is not their own to analyze, reflect on, and develop. The intent is to prompt student reflection with an incorrect, incomplete, or ambiguous written argument or explanation, and for students to improve upon the written work by correcting errors and clarifying meaning. Teachers can model how to effectively and respectfully critique the work of others with meta-think-alouds and press for details when necessary. This routine fortifies output and engages students in meta-awareness.

Example 1 – Critique a Partial or Flawed Response

1. PRESENT: Present a partial/broken argument, explanation, or solution method. Teacher can play the role of the student who produced the response, and ask for help in fixing it.
    - Given response could include a common error.
    - Given response should include an ambiguous term or phrase, or an informal way of expressing a mathematical idea.
2. PROMPT: Prompt students to identify the error(s) or ambiguity, analyze the response in light of their own understanding of the problem, and work both individually and in pairs to propose an improved response.
3. SHARE: Pairs share out draft improved response.
4. REFINE: Students refine their own draft response.

Example 2 – Always-Sometimes-Never

Use a structure or graphic organizer to evaluate or critique whether mathematical statements are always, sometimes, or never true. (Examples: 'A rectangle is a parallelogram' or 'A negative integer minus another negative integer equals a positive integer'.) Use the graphic organizer to frame and assess the reasoning process as students work toward evaluating and improving a response.

Mathematical Language Routine 4: Information Gap

Purpose: To create a need for students to communicate (Gibbons, 2002).

This routine allows teachers to facilitate meaningful interactions by giving partners or team members different pieces of necessary information that must be used together to solve a problem or play a game. With an information gap, students need to orally (and/or visually) share their ideas and information in order to bridge the gap and accomplish something that they could not have done alone. Teachers should model how to ask for and share information, clarification, justification, and elaboration. This routine cultivates conversation.

Example 1– Info Gap Cards

In one version of this activity, Partner A has the general problem on a card, and Partner B has the information needed to solve it on the "data card." Data cards can also contain diagrams, tables, graphs, etc. Partner A needs to realize what is needed and ask for information that is provided on Partner B's data card. Partner B should not share information unless Partner A specifically asks for it. Neither partner should read their cards to one another nor show their cards to their partners. As they work the problem, they justify their responses using clear and connected language.

1. READ, then THINK-ALOUD: The problem card partner (Partner A) reads his or her card silently and thinks aloud about what information is needed. Partner B reads the data card silently.
2. QUESTION 1: Partner B asks, "What specific information do you need?" Partner A needs to ask for specific information from Partner B.
3. QUESTION 2: When partner A asks, Partner B should ask for justification: "Why do you need that information?" before telling it to Partner A.
4. EXPLANATIONS: Partner A then explains how he or she is using the information to solve the problem. Partner B helps and asks for explanations, even if he or she understands what Partner A is doing.
5. FOLLOW-UP: As a follow-up step, have both students use blank cards to write their own similar problem card and data card for other pairs to use.

Example 2 – Info Gap Games

Students play a guessing game or matching game in which they have a real reason to talk (e.g., students need to work together to develop a strategy to win a game; each student is provided with different information; one student has something in mind and other students use their understanding of a

mathematical concept to guess what it is).
EXAMPLE: *Guess my ratio.* One student identifies a ratio between two distinct features/objects in the room (or within a given set of objects) and keeps the features/objects secret; other students try to figure out which features/objects are in the identified ratio.

Mathematical Language Routine 5: Co-Craft Questions and Problems

Purpose: To allow students to get inside of a context before feeling pressure to produce answers, to create space for students to produce the language of mathematical questions themselves, and to provide opportunities for students to analyze how different mathematical forms can represent different situations. Through this routine, students are able to use conversation skills to generate, choose (argue for the best one), and improve questions, problems, and situations as well as develop meta-awareness of the language used in mathematical questions and problems. Teachers should push for clarity and revoice oral responses as necessary.

Example 1 – Co-Craft Questions

1. PRESENT SITUATION: Teacher presents a situation – a context or a stem for a problem, with or without values included. (Example: A bird is flying at 30 mph)
2. STUDENTS WRITE: Students write down possible mathematical questions that might be asked about the situation. These should be questions that they think are answerable by doing math. They can also be questions about the situation, information that might be missing, and even about assumptions that they think are important. (1-2 minutes)
3. PAIRS COMPARE: In pairs, students compare their questions. (1-2 minutes)
4. STUDENTS SHARE: Students are invited to share their questions, with some brief discussion. (2-3 minutes)
5. REVEAL QUESTIONS: The actual questions students are expected to work on are revealed, and students are set to work.

Example 2 – Co-Craft Problems

1. PAIRS CREATE NEW PROBLEMS: Students get into pairs and co-create problems similar to a given task.
2. STUDENTS SOLVE THEIR OWN PROBLEMS: Students solve their own problems before trading them with other pairs.
3. EXCHANGE PROBLEMS: Students solve other pairs' problems, and check solutions and methods with the pair who created each problem.
4. TOPIC SUPPORT: Teacher can provide possible topics of interest to students, or brainstorm as a whole class for 2 minutes before pairing up.

Example 3 – Co-Craft Situations

1. PRESENT REPRESENTATION: Teacher presents a mathematical representation (graph, equation, function, table, etc.) with no labels.
2. STUDENTS WRITE: Students write stories or situations that correspond to the mathematical representation.
3. PAIRS COMPARE: Students explain how events in their partner's story or aspects of their partner's situation correspond to specific parts of the mathematical representation. They can ask clarifying questions or for more detail to do this.
4. REVISE STORIES: Students revise their stories adding details and clarification where needed.

Mathematical Language Routine 6: Three Reads

Purpose: To ensure that students know what they are being asked to do, create opportunities for students to reflect on the ways mathematical questions are presented, and equip students with tools used to negotiate meaning (Kelemanik, Lucenta & Creighton, 2016). This routine supports reading comprehension, sense-making, and meta-awareness of mathematical language. It also supports negotiating information in a text with a partner in mathematical conversation.

Example 1

Students are supported in reading a situation/problem three times, each time with a particular focus:

1. Students read the situation with the goal of comprehending the text (describe the situation without using numbers),
2. Students read the situation with the goal of analyzing the language used to present the mathematical structure.
3. Students read the situation in order to brainstorm possible mathematical solution methods.

This routine works well in conjunction with Mathematical Language Routine 5, in which the question stem is tentatively withheld in order to focus on the comprehension of what is happening in the text.

Example 2 – Values/Units Chart

Students use text-annotation to make sense of mathematical text using a two-column graphic organizer.

1. On the first read, students read through mathematical text and underline any words or phrases that represent a known or unknown value or amount. They list these numbers, unknowns, and variables in the left column of their graphic organizer (Values).
2. After the second read, in the right column (Units), students write the meaning of the value in context.
3. After the third read, students work in pairs to create mathematical expressions using only the right column. If they get stuck, encourage them to help press each other to make their right column descriptions more specific.

EXAMPLE: *It costs $3 per person to go to the Zoo. Alexandra's family has a coupon for a $5 discount. There are p people in Alexandra's family. Write an expression for how much it would cost for them to go to the zoo.*

Value (numerical or unknown)	*Units (reference to context)*
3	*$ per person to go to zoo*
5	*$ discounted from cost*
p	*number of people in family*
C	*cost for family to go to zoo*

*cost for family to go to zoo is ($ per person to go to zoo) * (number of people in family) - ($ discount)*
*C = 3 * p - 5*
C = 3p - 5

Mathematical Language Routine 7: Compare and Connect

Purpose: To foster students' meta-awareness as they identify, compare, and contrast different mathematical approaches, representations, concepts, examples, and language. Students should be prompted to reflect on and linguistically respond to these comparisons (e.g., exploring why or when one might do/say something a certain way, identifying and explaining correspondences between different mathematical representations or methods, wondering how an idea compares or connects to other ideas and/or language.) Teachers should model thinking out loud about these questions. This routine supports meta-cognitive and meta-linguistic awareness, and also supports mathematical conversation.

Example 1 – Compare and Connect Solution Strategies

Tell students their job is to understand one another's solution strategies by relating and connecting other students' approaches to their own approach.

1. SET-UP: Ways to set this up so that multiple strategies are likely to be generated by each pair of students:
   - I solve it one way, you solve it another
   - Divide and conquer: you do one and I do another
   - I have a piece of info, you have a piece of info
2. WHAT IS SIMILAR, WHAT IS DIFFERENT: Students first identify what is similar and what is different about the approaches. This can also be an initial discussion about what worked well in this or that approach, and what might make this or that approach more complete or easy to understand.
3. MATHEMATICAL FOCUS: Students are asked to focus on specific mathematical relationships, operations, quantities and values. For example:
   - Why does this approach include multiplication, and this one does not?
   - Where is the 10 in each approach?
   - Which unit rate was used in this approach?
   - Who can restate ___'s reasoning in a different way?"
   - "Did anyone solve the problem the same way, but would explain it differently?"
   - "Did anyone solve the problem in a different way?"
   - "Does anyone want to add on to _____'s strategy?"
   - "Do you agree or disagree? Why?"

Example 2 - Which One Doesn't Belong?

Pairs of students are provided with sets of four numbers, equations, expressions, graphs, or geometric figures. They must decide together how to group the sets so that three of the items fit within a category they have created and one does not. Both partners should be prepared to explain to a different group how they agreed on a category and justify which item did not fit.

Mathematical Language Routine 8: Discussion Supports

Purpose: To support rich and inclusive discussions about mathematical ideas, representations, contexts, and strategies (Chapin, O'Connor, & Anderson, 2009).

. The examples provided can be combined and used together with any of the other routines. They include multi-modal strategies for helping students make sense of complex language, ideas, and classroom communication. The examples can be used to invite and incentivize more student participation, conversation, and meta-awareness of language. Eventually, as teachers continue to model, students should begin using these strategies themselves to prompt each other to engage more deeply in discussions.

Example 1 – Whole Class Discussion Supports

- Revoice student ideas to model mathematical language use by restating a statement as a question in order to clarify, apply appropriate language, and involve more students.
- Press for details in students' explanations by requesting for students to challenge an idea, elaborate on an idea, or give an example.
- Show central concepts multi-modally by utilizing different types of sensory inputs: acting out scenarios or inviting students to do so, showing videos or images, using gesture, and talking about the context of what is happening.
- Practice phrases or words through choral response.
- Think aloud by talking through thinking about a mathematical concept while solving a related problem or doing a task. Model detailing steps, describing and justifying reasoning, and questioning strategies.

Example 2 – Numbered Heads Together

1. STUDENTS COUNT OFF – Each group of students count off by the number of students in the group so that every group has a 1, 2, 3, 4, etc.
2. POSE A QUESTION/PROBLEM – Teacher presents a question or problem that requires explanation or justification.
3. HEADS TOGETHER – Students have a certain amount of time to make sure that everyone in the group can explain or justify each step or part of the problem. They can create notes together during this stage.
4. REPORTING – Teacher calls a random number from 1-4. At that point, groups are no longer allowed to talk or write to each other but the reporters are allowed to use the notes that have already been created. The students with the number called are the reporters for their group. The teacher asks the reporters, one at a time, to explain the next step of the problem, to agree/disagree with the previous reporter, or to justify the reasoning of their group in some way. Correct answers are not revealed or agreed upon until every reporter shares.

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